

Storage Life Evaluation of Optocoupler under Competitive Failure Condition

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Abstract:

Purpose: For assessing the storage life of the terminal-guided projectile optocoupler, the reliability of two-factor independent competition failure based on Wiener process-based degradation failure and Weibull-based sudden failure is carried out. **Methods:** In the aspect of degradation failure modeling, the linear Wiener process is improved. The drift coefficient is randomized by establishing the random variable Arrhenius equation. The problem of transforming nonlinear data into linear data is solved by using the time-scale transformation formula, and parameter estimates were solved by the two-step maximum likelihood estimation method. The Weibull distribution is studied for sudden failure modeling. Assuming $m < 10$ and $t_{0.9} > 10$, the “data island” method is proposed. The obtained convergence solution was evaluated by the maximum likelihood estimation, and then the initial values of three parameters are determined. **Results:** Its $t_{0.9}$ is about 25.46 to 27.5 years in the case of degradation failure only and 17.1 years in the case of sudden failure only. **Conclusion:** The long-term storage of the optocoupler is estimated to be about 17 years under 0.9 reliability conditions due to the combination of degradation failure and burst failure.

Keywords: *Optocoupler; Storage reliability; Wiener process; Random variable Arrhenius equation; Weibull distribution; Data island.*

I. INTRODUCTION

As an important military material, ammunition is different from other weapons and equipment, and it has the important characteristics of “long-term storage, one-time use” [1]. Therefore, research on the reliability and storage life of ammunition, especially informational ammunition storage, is crucial. At present, informational ammunition has large reserves and high value, but over time, its life assessment problem has become increasingly prominent, and the large-scale use of optoelectronic devices on informational ammunition has determined that its

storage life is quite different from that of ordinary traditional ammunition. Therefore, research on the storage reliability technology of informational ammunition optoelectronic devices has significant military application value and extensive economic benefits. From the related work at home and abroad, when discussing the storage life of informational ammunition, the storage life of the weak link must be first considered.

Through the detection of a large number of missiles with a near-warranty period, it is found that the missile's internal optocoupler has serious quality problems. On the contrary, some traditional mechanical components have better storage reliability than electronic components represented by optocouplers. In order to evaluate the storage life of the optocoupler inside the control cabin of the terminal guided projectile, 10 samples were selected and subjected to SSADT. The stress levels are 70 °C, 90 °C, 110 °C and 120 °C. The number of tests was 23, 11, 10 and 9 times. The test interval is 8 hours, and its total test time is 424 hours. Therefore, the moments of stress conversion are 0th hour, 184th hour, 272th hour, 352th hour and 424th hour, respectively. According to previous research, the optocouplers have two independent failure modes, namely, sudden failure and leakage current degradation failure. In the formal test, both failure modes appeared. Among the 10 samples, 6 optocouplers' leakage current have degraded, and the remaining 4 optocouplers have not degraded, but 2 optocouplers have sudden failures after heating 408 and 424 hours respectively.

According to the previous research, it is assumed that product degradation failure and sudden failure are independent. So the reliability function of the optocoupler at time t can be obtained as the product of the reliability function of the degradation failure and the reliability function of the sudden failure [2]. The current problem is transformed into how to obtain the reliability function under the condition of sudden failure and degradation failure. Here first discuss the reliability function under degraded failure condition, and then discuss the reliability function under sudden failure condition.

II. CONSTRUCTION OF ACCELERATION MODEL BASED ON RANDOM VARIABLES

2.1 The Focus of Leakage Current Wiener Process Modeling

Taking the leakage current parameter of the optocoupler as the research object, the accelerated degradation model was carried out to evaluate its long-term storage reliability. At this stage, there are generally three methods for performance degradation modeling: performance degradation orbit method, degradation quantity distribution method, and stochastic process method [3, 4]. The first two methods are simpler, most widely used, and mature in technology. However, the drawback of both is that it ignores the randomness of the samples during degradation. Compared with the degraded orbital method or the degenerate quantity distribution method, the stochastic process-based method takes into account the randomness and dynamic characteristics of the degraded process, which can better reflect the comprehensive impact of environmental factors on product performance [5]. There are three main stochastic processes currently used to construct

degenerate models: the Wiener process, the composite Poisson process, and the gamma process. Their main difference between Wiener process and Gamma process is that the former can describe the case where the delta increment may be negative, while the gamma process requires that the delta increment is non-negative [6,7]. The composite Poisson process is usually used to describe discrete degradation processes [8]. Among the three, the Wiener process is most widely used [9]. Considering that the experimental data degradation increment of the optocoupler may be negative, it is determined that the Wiener process is used for the optocoupler's leakage current degradation data .

Assuming that the degradation process of the optocoupler leakage current can be described as follow [10]:

$$X(t) = x_0 + \lambda t + \sigma_w W(t) \tag{1}$$

In the above formula, x_0 is the initial value of the leakage current, λ is the drift parameter, which characterizes the degradation rate, and σ_w is the diffusion parameter, and $W(t)$ is the standard Wiener process.

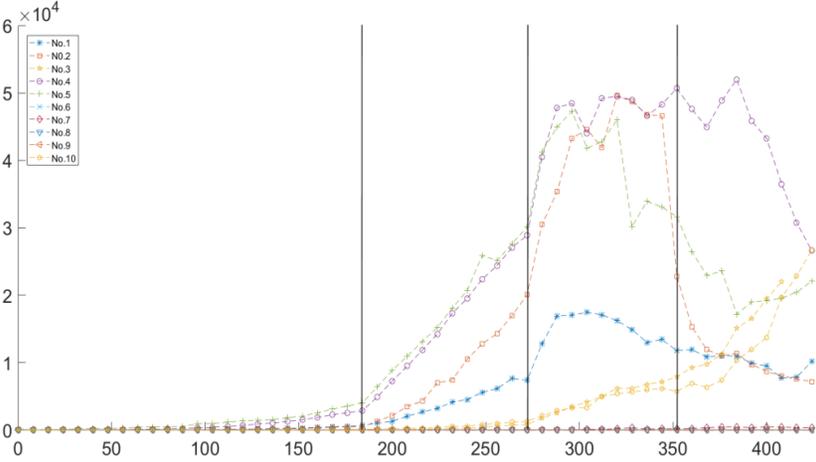


Fig 1: Leakage current with time

Figure 1 is scatter plots of the raw measurement data for the leakage current of No. 1 to No. 10 optocouplers over time. The vertical lines in the figure represent the division of different stress stages. The horizontal axis represents time in hours. The vertical axis represents the measured value in nA. Since only 6 optocoupler leakage currents are degraded, the remaining four leakage currents do not change. Therefore, this paper discusses the reliability modeling of six optocouplers with leakage current changes. The Wiener process is widely used in the application, but when modeling the data of leakage current, it needs to face the following two key issues:

- (1) How to describe the individual differences in leakage current degradation of different optocouplers;
- (2) Processing of leakage current nonlinear data.

The same batch of optocouplers showed individual differences in the rate of degradation of leakage current due to factors such as manufacturing process, design error, and environment and

materials. In the formal test process, it can be seen from Figure 1 that this phenomenon is particularly prominent. Li considers x_0 and λ as random variables when evaluating the reliability of the satellite momentum wheel, and then evaluates the reliability of the momentum wheel [11]. However, the momentum wheel data is collected under non-accelerated test conditions, that is, real-time monitoring data, and does not involve acceleration problems. Both Wang and Guo regard λ and σ_w as random variables, but the assumed distribution type has not been tested, and the EM algorithm given by it is computationally complex and not suitable for generalization [12, 13]. Cai and Tang regard λ as a random variable obeying normal distribution, and give a life model considering individual differences, which is convenient for calculation and achieve good model fitting effect [14, 15]. At present, the processing of nonlinear data mainly adopts the time scale transformation model which was first proposed by Whitmore [16]. Many literatures use its model, and the examples prove that it has good practicability [17]. Based on the above analysis, the leakage current degradation data is first time-scale transformed to make the nonlinear data of the leakage current change into linear data, and then λ is randomized to construct an accelerated degradation model considering individual differences, then the unknown parameters are estimated by maximum likelihood estimation method. Since x_0 is almost negligible compared to the failure threshold, and x_0 is more concentrated, only one point is outside the concentrated area, so x_0 is averaged instead of x_0 as a random variable if it is treated as a random variable. Not only is it unnecessary but also greatly increases the complexity of parameter estimation [18].

2.2 Acceleration Model Based on Random Variables

The acceleration equation of the optocoupler leakage current satisfies the Arrhenius acceleration model. It is assumed that λ is related to the stress, the corresponding acceleration model is:

$$\lambda_i = a \exp(-b/T_i) \quad (2)$$

In the above formula, a and b are unknown constants, and T_i is an absolute temperature. According to the above formula, the degradation rate is the same at each stress level. As mentioned above, the individual differences determine the inconsistency of the rate of degradation of different optocoupler leakage currents, and the error will occur with the use of deterministic parameters. Therefore, the acceleration equation of the j -th optocoupler under the i -th stress is:

$$\lambda_i^j = a_j \exp(-b/T_i), a_j \sim N(\mu_a, \sigma_a^2) \quad (3)$$

It can be seen from the above formula that the drift coefficient λ_i considering the difference in the degradation of the optocoupler under the i -th stress can be expressed as:

$$\lambda_i \sim N(\mu_a \exp(-b/T_i), \sigma_a^2 \exp(-b/T_i)) \quad (4)$$

When σ_a^2 is 0, the random variable Arrhenius model returns to the traditional model [19].

III. DATA DESCRIPTION BY WIENER PROCESS AND MODELING OF NONLINEAR DEGRADATION DATA UNDER STEP STRESS CONDITION

3.1 Statistical Model of SSADT Data

If there are n optocouplers performing step test under l stresses, the k_i times data ($i = 1, 2, \dots, l$) are measured under each stress, and the total number of measurements is $K = \sum_{i=1}^l k_i$. Then the measurement moment of the j -th optocoupler under the i -th stress is $t_{i,k_i^q}^j$ ($i=1, 2, \dots, l, q=1, 2, \dots, Q, Q=k_i$), and the amount of performance degradation is $X(t_{i,k_i^q}^j)$. It can be seen from the step test that the initial value of the degradation amount of the j -th photocoupler under the i -th stress is the end value of the degradation amount under the $(i-1)$ -th stress, that is, $X(t_{i,0}^j) = X(t_{i-1,k_{i-1}^Q}^j)$. In view of this, for each optocoupler leakage current data model is established as follows:

$$X(t_{i,k_i^q}^j) = \begin{cases} x_0 + \lambda_1 t_{i,k_i^q} + \sigma_w W(t_{i,k_i^q}) & 0 \leq t_{i,k_i^q} < t_{1,k_1^Q} \\ x_0 + \lambda_2 (t_{i,k_i^q} - t_{1,k_1^Q}) + \lambda_1 t_{1,k_1^Q} + \sigma_w W(t_{i,k_i^q}) & t_{2,0} \leq t_{i,k_i^q} < t_{2,k_2^Q} \\ \dots\dots\dots \\ x_0 + \lambda_l (t_{i,k_i^q} - t_{l-1,k_{l-1}^Q}) + \sum_{i=1}^{l-1} \lambda_i (t_{i,k_i^Q} - t_{i-1,k_{i-1}^Q}) + \sigma_w W(t_{i,k_i^q}) & t_{l,0} \leq t_{i,k_i^q} < t_{l,k_l^Q} \end{cases} \quad (5)$$

Under the i -th stress, the j -th optocoupler has the performance degradation increment at the q -th measurement compared to the $(q-1)$ -th measurement:

$$\Delta X(t_{i,k_i^q}^j) = X(t_{i,k_i^q}^j) - X(t_{i,k_i^{q-1}}^j) \quad (6)$$

The time interval is:

$$\Delta t_{i,k_i^q}^j = t_{i,k_i^q}^j - t_{i,k_i^{q-1}}^j \quad (7)$$

By the nature of the Wiener process:

$$\Delta X(t_{i,k_i^q}^j) \sim N(\lambda_i \Delta t_{i,k_i^q}^j, \sigma_B^2 \Delta t_{i,k_i^q}^j) \quad (8)$$

3.2 Model of Nonlinear Degradation Data

For nonlinear data, the time scale transformation model proposed by Whitmore is used to transform it into linear data. Its common functions are:

$$\tau = \Lambda(t) = t^c \quad (9)$$

In the above formula, c is a constant and greater than zero. When $c < 1$, the data is convexly degraded, and when $c > 1$, the data is concavely degraded [20]. Both of these data are non-linear data. When $c = 1$, the data is linear degradation, that is, linear data. When more and more nonlinear data appears in the data, the value of c begins to deviate slightly from 1, and the more nonlinear data, the greater the deviation of c value. The advantage of the above equation is that it can handle the case where linear and nonlinear data coexist.

Therefore, this paper uses the above transformation model to rewrite the leakage current data from $[t, X(t)]$ to $[\tau, Y(\tau)]$, so we can rewrite (1) as:

$$Y(\tau) = x_0 + \lambda\tau + \sigma_w W(\tau) \quad (10)$$

If failure threshold is D_f , the lifetime reliability function of leakage current based on the Wiener process is:

$$R(t) = \Phi\left(\frac{D_f - \lambda t}{\sigma_w \sqrt{t}}\right) - \exp\left(\frac{2\lambda D_f}{\sigma_w^2}\right) \Phi\left(\frac{-D_f - \lambda t}{\sigma_w \sqrt{t}}\right) \quad (11)$$

In the above formula, $\Phi(\cdot)$ is a standard normal distribution function. After a simple derivation, the transformed reliability function can be obtained as follows:

$$R(\tau) = \Phi\left(\frac{D_f - \lambda\tau}{\sigma_w \sqrt{\tau}}\right) - \exp\left(\frac{2\lambda D_f}{\sigma_w^2}\right) \Phi\left(\frac{-D_f - \lambda\tau}{\sigma_w \sqrt{\tau}}\right) \quad (12)$$

Order $\tau = \Lambda(t)$ and $Y(\tau) = X(t)$, then the above formula can be turned into:

$$R(t) = R(\Lambda(t)) = \Phi\left(\frac{D_f - \lambda\Lambda(t)}{\sigma_w \sqrt{\Lambda(t)}}\right) - \exp\left(\frac{2\lambda D_f}{\sigma_w^2}\right) \Phi\left(\frac{-D_f - \lambda\Lambda(t)}{\sigma_w \sqrt{\Lambda(t)}}\right) \quad (13)$$

As described above, in order to reflect the difference in the degradation of the optical coupler, λ is randomized, and λ can be expressed in the form of equation (4). Let the λ mean be μ_λ and the variance be σ_λ^2 , then the above formula can be turned into:

$$R_T(t) = \Phi\left(\frac{D_f - \lambda\Lambda(t)}{\sigma_w \sqrt{\sigma_w^2 + \sigma_\lambda^2 (\Lambda(t))^2}}\right) - \exp\left[\frac{2\lambda D_f}{\sigma_w^2} + \frac{2\sigma_\lambda^2 D_f^2}{\sigma_w^4}\right] \bullet \Phi\left(\frac{2\sigma_\lambda^2 D_f + \sigma_w^2 (D_f + \mu_\lambda \Lambda(t))}{\sigma_w^2 \sqrt{\sigma_w^2 \Lambda(t) + \sigma_\lambda^2 (\Lambda(t))^2}}\right) \quad (14)$$

The above formula is the most concerned reliability function. Since q is related to stress, it is strictly a binary function. When the stress is given, such as constant stress, it becomes a unitary function [21]. So the next step is to estimate the relevant parameters in the above formula.

IV. SOLUTION OF RELIABILITY FUNCTION

4.1 Two-step Maximum Likelihood Estimation

In the foregoing, the leakage current data $[t, X(t)]$ is converted into $[\tau, Y(\tau)]$, so the equation (8) can be transformed into:

$$\Delta Y\left(\tau_{i,k^q}^j\right) \sim N\left(\lambda_i^j \Delta \tau_{i,k^q}^j, \sigma_w^2 \Delta \tau_{i,k^q}^j\right) \quad (15)$$

In the above formula,

$$\begin{cases} \Delta Y\left(\tau_{i,k^q}^j\right) = Y\left(\tau_{i,k^q}^j\right) - Y\left(\tau_{i,k^{q-1}}^j\right) \\ \Delta \tau_{i,k^q}^j = \tau_{i,k^q}^j - \tau_{i,k^{q-1}}^j \\ \tau_{i-1,k_{i-1}^0}^j = \tau_{i,0}^j \\ \tau_{1,0}^j = 0 \\ i = 1, 2, \dots, l; j = 1, 2, \dots, n; q = 1, 2, \dots, Q; Q = k_i \end{cases} \quad (16)$$

Where $\Delta\tau_{i,k^q}^j$ is the incremental value of the time change of the q -th measurement of the j -th photocoupler under the i -th stress, and $\Delta Y(\tau_{i,k^q}^j)$ is the corresponding degradation increment. According to the above two formulas, the maximum likelihood estimation function is established:

$$\ln L = -\frac{nK}{2}(\ln(2\pi) + \ln\sigma_w^2) - \frac{n}{2} \sum_{i=1}^l \sum_{q=1}^Q \ln[\Delta\tau_{i,k^q}^j] - \frac{1}{2\sigma_w^2} \sum_{j=1}^n \sum_{i=1}^l \sum_{q=1}^Q (\Delta Y(\tau_{i,k^q}^j) - \lambda_i^j \Delta\tau_{i,k^q}^j) / \Delta\tau_{i,k^q}^j \quad (17)$$

Substituting $\tau = t^c$ and $\Delta Y(\tau_{i,k^q}^j) = \Delta X(t_{i,k^q}^j)$ and formula (4) into the above formula, we can get:

$$\begin{aligned} \ln L = & -\frac{nK}{2}(\ln(2\pi) + \ln\sigma_w^2) - \frac{n}{2} \sum_{i=1}^l \sum_{q=1}^Q \ln \left[(t_{i,k^q}^j)^c - (t_{i,k^{q-1}}^j)^c \right] \\ & - \frac{1}{2\sigma_w^2} \sum_{j=1}^n \sum_{i=1}^l \sum_{q=1}^Q \left\{ \Delta X(t_{i,k^q}^j) - a_j \exp(-b/T_i) \left[(t_{i,k^q}^j)^c - (t_{i,k^{q-1}}^j)^c \right] \right\}^2 / \left[(t_{i,k^q}^j)^c - (t_{i,k^{q-1}}^j)^c \right] \end{aligned} \quad (18)$$

Let the first-order partial derivatives of a_j and σ_w^2 of the above formula be 0, that is,

$$\begin{cases} \frac{\partial \ln L}{\partial a_j} = 0 \\ \frac{\partial \ln L}{\partial \sigma_w^2} = 0 \end{cases} \quad (19)$$

It can be found:

$$\hat{a}_j = \frac{\sum_{i=1}^l \sum_{q=1}^Q \Delta X(t_{i,k^q}^j) \exp(-b/T_i)}{\sum_{i=1}^l \sum_{q=1}^Q \exp(-2b/T_i) \left[(t_{i,k^q}^j)^c - (t_{i,k^{q-1}}^j)^c \right]} \quad (20)$$

$$\hat{\sigma}_w^2 = \frac{1}{nK} \sum_{j=1}^n \sum_{i=1}^l \sum_{q=1}^Q \left\{ \Delta X(t_{i,k^q}^j) - a_j \exp(-b/T_i) \left[(t_{i,k^q}^j)^c - (t_{i,k^{q-1}}^j)^c \right] \right\}^2 / \left[(t_{i,k^q}^j)^c - (t_{i,k^{q-1}}^j)^c \right] \quad (21)$$

Theoretically, equations (20) and (21) can be solved with degenerate data, but carefully observe the above two equations, which contain b and c , so the two-step maximum likelihood estimation method is used here [22].

The first step: the estimation of \hat{b} and \hat{c} . Both were estimated using the *fminsearch* function in Matlab software. But this function solves the minimum point, so a negative sign before the equation is used. Substitute equations (20) and (21) into equation (18), assign initial values to b and c , and then perform a two-dimensional traversal search to obtain estimates of \hat{b} and \hat{c} .

The second step: Substituting the obtained estimates of \hat{b} and \hat{c} into equations (20) and (21), an estimate of \hat{a}_j and $\hat{\sigma}_w^2$ ($j = 1, 2, \dots, 6$) can be obtained. In addition, $\hat{\mu}_a$ and $\hat{\sigma}_a^2$ can also be obtained by the following formula:

$$\begin{cases} \hat{\mu}_a = \frac{1}{n} \sum_{j=1}^n a_j \\ \hat{\sigma}_a^2 = \frac{1}{n} \sum_{j=1}^n (a_j - \hat{\mu}_a)^2 \end{cases} \quad (22)$$

After $\hat{\mu}_a$ and $\hat{\sigma}_a^2$ are obtained, according to equation (4), the mean and variance of λ under given stress conditions can be known. Therefore, the reliability function of the photocoupler leakage current degradation failure can be obtained by substituting it into equation (14).

4.2 Threshold and Reliability Curves

Through the previous derivation and established model, given $n=6, l=4, k_1=23, k_2=11, k_3=10, k_4=9, T_1=70+273.15, T_2=90+273.15, T_3=110+273.15, T_0=25+273.15$. Combined with the measured data, the parameters such as the drift coefficient and the diffusion coefficient can be calculated.

TABLE I. Estimated values of various parameters

\hat{b}	\hat{c}	$\hat{\mu}_\lambda / 10^{-4}$	$\hat{\sigma}_\lambda^2 / 10^{-8}$	$\hat{\sigma}_w^2 / 10^{-4}$
3298.1	1.4951	3.6292	2.3819	4.3258

The $\hat{\mu}_\lambda$ and $\hat{\sigma}_\lambda^2$ in the above table refer to the estimated values under normal temperature stress. By substituting the parameter values into equation (14), the concrete expression of the reliability function can be obtained.

The concern now is the failure threshold D_f . According to the failure mechanism verification test, the optocoupler enters an unstable state when $D_f=70$ microamps, but at that time, the optocoupler has already produced a substantial failure, that is, the predetermined function cannot be completed. By communicating with optocoupler manufacturers, manufacturers can't give a clear statement, because there is no in-depth study of the storage reliability of the product, so it can not provide a specific failure threshold. Therefore, the sliding threshold is set, starting from 60 microamps, and 2 microamps is the step size up to 70 microamps. Its reliability curve is shown in Figure 2.

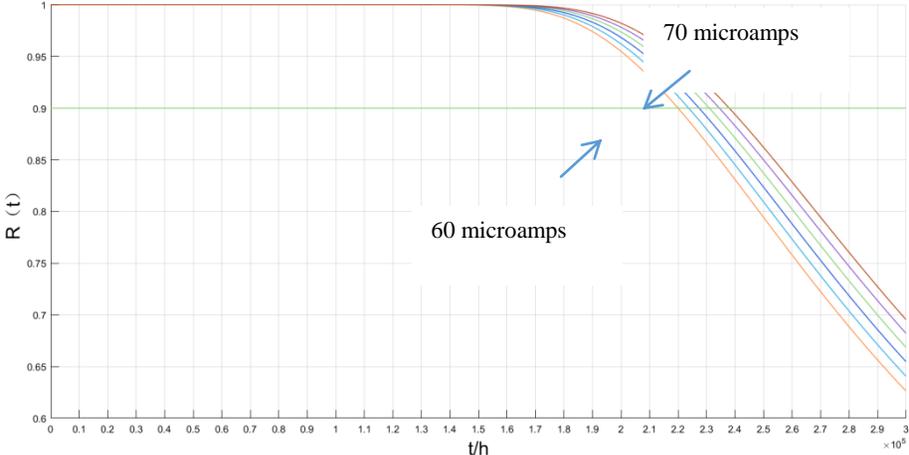


Fig 2: Optocoupler Wiener process reliability curve based on sliding threshold

As shown in the above figure, the lowest reliability curve is the reliability curve when the failure threshold is 60 μA , which is arranged in order, and the reliability curve at the top is 70 μA .

As the threshold increases, it can be seen that $t_{0.9}$ continues to increase, with $t_{0.9}$ ranging from approximately 220,000 hours (approximately 25.46 years) to 237,600 hours (approximately 27.5 years).

V. RELIABILITY ASSESSMENT UNDER SUDDEN FAILURE CONDITIONS

5.1 Description of Failure Data

Now consider the storage reliability of optocouplers in the event of a sudden failure. The stress profile and optocoupler failure are shown in Figure 3.

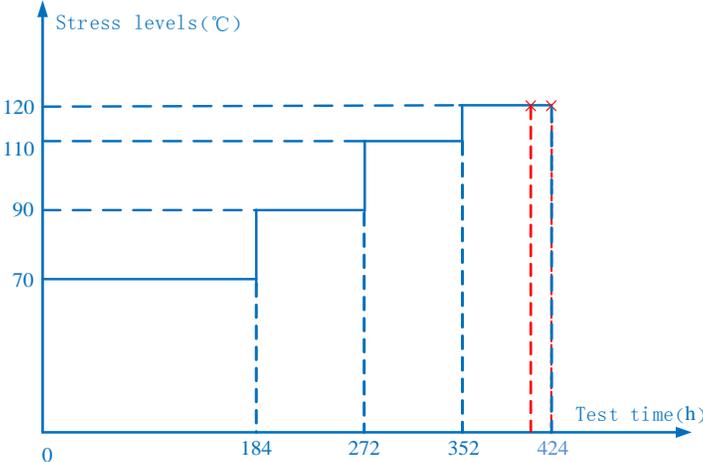


Fig 3: Test profile and optocoupler failure nodes

Assume that the formal test of the optocoupler has a total of n stress levels, $S_0 < S_1 < \dots < S_i < \dots < S_n$ ($i=1, 2, 3, \dots, n$). S_0 is the stress level at room temperature (25°C). The number of samples that fail in stress S_i is r_i , and the failure time is:

$$0 < t_{i1} \leq t_{i2} \leq \dots \leq t_{ir_i} \leq \tau_i \tag{23}$$

τ_i is the truncation point at the stress S_i , that is, the moment when the stress changes. Suppose k samples are in the test. The total number of failed samples is $r = \sum_{i=1}^n r_i$, and the number of samples that have not failed at the time τ_n of the test truncation is $k-r$. Obviously, the failed data of the test belongs to the timing censoring type data. According to the actual situation, $k=10$, $n=4$, and $r=2$.

5.2 Reliability Based on Weibull Distribution under Sudden Failure Conditions

As mentioned above, a total of 2 out of 10 optocouplers have a sudden failure. According to the previous experience, the sudden failure life function of this kind of optocoupler is Weibull distribution. So its distribution function is [23]:

$$F_i(t) = 1 - \exp \left[- \left(\frac{t}{\eta_i} \right)^{m_i} \right], i = 1, 2, \dots, n. \quad (24)$$

Where η_i and m_i are the scale and shape parameters under stress S_i , respectively. The following three basic assumptions for statistical analysis using the Weibull distribution are given below:

- 1 The shape parameters under each stress level S_i are equal, that is, $m_0 = m_1 = \dots = m_n = m$.
- 2 The characteristic life η_i and stress S_i of the product satisfy the Arrhenius acceleration equation [24]:

$$\ln \eta_i = a + b \cdot \varphi(S_i). \quad (25)$$

Where $a = \ln A$ and $b = -E_a/R$ are unknown parameters, $\varphi(S_i) = 1/S_i = 1/T_i$, $i = 1, 2, \dots, n$, and T_i is the absolute temperature.

- 3 The residual life of the product depends only on the cumulative failure amount and the stress level at that time, and is independent of the cumulative mode (Nelson cumulative failure assumption).

Since the formal test is a step test, except for the "true life" of the failure time in the first stress stage, the "life time" under the stress of the other stages needs to be converted. Here, the "life time" is converted to the first stress phase, that is, 70°C , and the first stress phase is represented by S_h . According to the definition in the literature, the acceleration factors $Q_{i,h}$ of stress S_i and S_h is:

$$Q_{i,h} = \frac{\eta_i}{\eta_h} = \exp \left[\frac{E_a}{R} \left(\frac{1}{T_i} - \frac{1}{T_h} \right) \right] = \exp \left[b \left(\frac{1}{T_h} - \frac{1}{T_i} \right) \right]. \quad (26)$$

The failure time of the j -th failure sample under stress S_i is converted to the failure time at a given stress S_h :

$$t_{ij}^h = Q_{i,h} \cdot t_{ij} + \sum_{l=1}^{i-1} Q_{l,h} \cdot \tau_l. \quad (27)$$

For those samples that have not failed at the test truncation time τ_n , the time to convert to a given stress level S_h is:

$$\tau^h = \sum_{l=1}^n K_{l,h} \cdot \tau_l. \quad (28)$$

Among them, $i = 1, 2, \dots, n$, $r = \sum_{i=1}^n r_i$, $j = 1, 2, \dots, r_i$, $K_{l,h}$ is the acceleration factor.

So the likelihood function can be derived as follows:

$$\begin{aligned} L(b, \eta_h, m) &= \prod_{i=1}^n \prod_{j=1}^{r_i} f(t_{ij}^h) \cdot [1 - F(\tau^h)]^{n-r} \\ &= \prod_{i=1}^n \prod_{j=1}^{r_i} \frac{m}{\eta_h} \left(\frac{t_{ij}^h}{\eta_h} \right)^{m-1} \exp \left[- \left(\frac{t_{ij}^h}{\eta_h} \right)^m \right] \cdot \exp \left[- (n-r) \left(\frac{\tau^h}{\eta_h} \right)^m \right]. \end{aligned} \quad (29)$$

Find the logarithm of the above formula, we can get:

$$\ln L(b, \eta_h, m) = \sum_{i=1}^n \sum_{j=1}^{r_i} \left[\ln m - \ln \eta_h + (m-1)(\ln t_{ij}^h - \ln \eta_h) - \left(\frac{t_{ij}^h}{\eta_h} \right)^m \right] - (n-r) \left(\frac{\tau^h}{\eta_h} \right)^m. \quad (30)$$

By solving partial differential equations:

$$\frac{\partial \ln L}{\partial b} = \frac{\partial \ln L}{\partial \eta_h} = \frac{\partial \ln L}{\partial m} = 0. \quad (31)$$

Maximum likelihood estimates of unknown parameters $(\hat{b}, \hat{\eta}_h, \hat{m})$ in the Weibull distribution can be obtained. The estimated value of the characteristic lifetime η_0 of the optocoupler at normal temperature stress levels is [25]:

$$\hat{\eta}_0 = \hat{\eta}_h \cdot \exp \left[\hat{b} \left(\frac{1}{T_0} - \frac{1}{T_h} \right) \right]. \quad (32)$$

Further, at a given reliability R , the estimated lifetime t_R of the product at a normal stress level is:

$$\hat{t}_R = (-\ln R)^{1/\hat{m}} \cdot \hat{\eta}_0 = (-\ln R)^{1/\hat{m}} \cdot \hat{\eta}_h \cdot \exp \left[\hat{b} \left(\frac{1}{T_0} - \frac{1}{T_h} \right) \right]. \quad (33)$$

Through the above derivation, the required reliability information is obtained. The question now becomes how to solve the partial differential equations.

VI. PARAMETER ESTIMATION BASED ON “DATA ISLAND” METHOD

6.1 Specific Steps for the “Data Island” Method.

The previous section gives a specific formula for how to solve the parameter estimates in the Weibull distribution. Since the failure data is too small (<3), the approximate estimate of the parameters are not known. This section offers a new way for how to solve parameters in the Weibull distribution relatively accurately in the absence of prior information of the parameter estimates. In general, the equations of (31) are currently solved using math software. Since the equations of (31) have a transcendental equation, they cannot be solved by the elementary method. The math software solves the equations using the Newton-Raphson method. This method requires an initial value to be set for the parameter being solved. Although the Newton-Raphson method has a second-order convergence rate, it is highly dependent on the selected initial value [26]. If the initial value is not set properly, it directly affects the result of the solution [27]. The disadvantage of the Newton-Raphson method is that it is too dependent on initial values. Once when selected initial values are too far from the analytical solution, the convergence result will not be obtained. The question now is how to choose a reliable initial value so that the results are as accurate as possible while satisfying convergence. Since there is no relatively reliable initial value estimation in the optocoupler test, a simple initial value acquisition method is studied and the

method is called “Data Island”. When using the "data island" approach, we need to first acknowledge two assumptions:

- (1) The missile control cabin has at least 10 years of life at 0.9 reliability.
- (2) The m is not greater than 10.

Essentially, these two assumptions belong to the added prior information. Without these two prior information, it is impossible to accurately assess the storage life of the optocoupler in the event of a sudden failure.

Based on these two assumptions, the specific steps of the “data island” method are given.

1). Determine how sensitive the three parameters (b, η_h, m) are to the solution. The method is as follows: arbitrarily fix two parameters, make one parameter change, and observe the severity of the change of the optocoupler $t_{0.9}$ reliability value. When fixed $b = \eta_h = 1$, make m change.

TABLE II. Estimated value of $t_{0.9}$ for different m (unit: year)

	$m=1$	$m=2$	$m=3$	$m=4$	$m=5$	$m=6$	$m=7$	$m=8$	$m=9$	$m=10$
$t_{0.9}$	4.049 9×10^{-11}	1.950 9×10^{-11}	2.579 9×10^{-11}	4.395 8×10^{-11}	2.315 7×10^{-11}	7.199 2×10^{-12}	2.635 7×10^{-16}	6.457 8×10^{-15}	5.433 6×10^{-16}	5.807 2×10^{-20}

When fixed $m = \eta_h = 1$, make b change.

TABLE III. Estimated value of $t_{0.9}$ for different b (unit: year)

	$b=1$	$b=100$	$b=1000$	$b=3000$	$b=6000$	$b=10000$	$b=30000$	$b=60000$	$b=90000$
$t_{0.9}$	4.049 9×10^{-11}	4.197 3×10^{-11}	4.457 9×10^{-11}	7.779 3×10^{-11}	1.439 7×10^{-10}	2.581 6×10^{-12}	1.380 7×10^{-69}	8.564 2×10^{-75}	5.555 1×10^{-96}

When fixed $b = m = 1$, make η_h change.

TABLE IV. Estimated value of $t_{0.9}$ for different η_h (unit: year)

	$\eta_h=1$	$\eta_h=100$	$\eta_h=1000$	$\eta_h=3000$	$\eta_h=6000$	$\eta_h=10000$	$\eta_h=30000$	$\eta_h=60000$	$\eta_h=90000$
$t_{0.9}$	4.049 9×10^{-11}	8.690 8.690×10^{-10}	0.246 4	0.031	0.031 2	0.031 4	0.033	0.033 9	0.044 1

It can be seen from the above three tables that when the other two are unchanged, when the value of m changes, there is a difference of 9 orders of magnitude in $t_{0.9}$, and when the value of b

changes, there is a difference of 85 orders of magnitude in $t_{0.9}$. When the value of η_h changes, there is a difference of 10 orders of magnitude between $t_{0.9}$. Therefore, the sensitivity of the initial values to the results is ranked as:

$$b > \eta_h > m$$

2). Preliminary determination of the range of values for each parameter. In theory, the wider the range of values, the better the establishment of the “data island”. However, if the ranges of the values are too large, the workload will increase sharply, the difficulty and time of the operation will increase, and the narrow ranges may miss the appropriate initial value. Therefore, it is necessary to comprehensively consider various factors to select an appropriate range of each value. Considering the engineering practice, for the sake of conservatism, the range of values should be appropriately relaxed. The range of b is [1, 15000], the range of η_h is [1, 15000], and the step is 1000, where the unit of η_h is hour.

3). Establishment of “Data Island”. Set the initial value of m to 10. When m is given, in steps of 1000, all the values of $t_{0.9}$ corresponding to the initial values of the ranges of b and η_h are calculated using math software. Then create a three-dimensional map with b and η_h as the X and Y axes and $t_{0.9}$ as the Z axis. In order to save space, all calculation results are not given. Intuitively, some calculation results are given here. For simplicity, note η_h is η , and the estimated \hat{b} and $\hat{\eta}$ are of the order of magnitude 10^3 , the remaining parameters are of the order of 10^0 , and the unit of $t_{0.9}$ is still a, ie year.

TABLE V. Part estimated values of parameters under different b and η conditions when $m=10$ (unit: year)

$\eta \backslash b$	1	1000	2000	3000	...	15000
1	$\hat{b} = -5.1869$ $\hat{m} = 0.1215$ $\hat{\eta} = 0.498$ $t_{0.9} = 5 \times 10^{-12}$	$\hat{b} = 3.6922$ $\hat{m} = 2.433$ $\hat{\eta} = 2.0647$ $t_{0.9} = 0.496$	$\hat{b} = -2.6458$ $\hat{m} = 1.1724$ $\hat{\eta} = 2.0067$ $t_{0.9} = 0.0104$	$\hat{b} = -1.0158$ $\hat{m} = 1.0416$ $\hat{\eta} = 3.0465$ $t_{0.9} = 0.0258$...	$\hat{b} = -0.7149$ $\hat{m} = 0.8484$ $\hat{\eta} = 5.1366$ $t_{0.9} = 0.0304$
1000	$\hat{b} = 0.992$ $\hat{m} = 0.0423$ $\hat{\eta} = 0.0047$ $t_{0.9} = 6 \times 10^{-27}$	$\hat{b} = 3.4096$ $\hat{m} = 2.6$ $\hat{\eta} = 1.8048$ $t_{0.9} = 0.4111$	$\hat{b} = 1.6064$ $\hat{m} = 1.6$ $\hat{\eta} = 1.6$ $t_{0.9} = 0.1266$	$\hat{b} = 0.2763$ $\hat{m} = 1.1534$ $\hat{\eta} = 2.9694$ $t_{0.9} = 0.0553$...	$\hat{b} = 0.2624$ $\hat{m} = 0.9025$ $\hat{\eta} = 5.0488$ $t_{0.9} = 0.0543$
2000	$\hat{b} = 1.8705$ $\hat{m} = 0.0409$ $\hat{\eta} = 0.0047$ $t_{0.9} = 2 \times 10^{-27}$	$\hat{b} = 4.3575$ $\hat{m} = 2.8444$ $\hat{\eta} = 2.1095$ $t_{0.9} = 0.7806$	$\hat{b} = 3.5469$ $\hat{m} = 1.9509$ $\hat{\eta} = 2.5027$ $t_{0.9} = 0.4482$	$\hat{b} = 1.283$ $\hat{m} = 1.2833$ $\hat{\eta} = 2.8833$ $t_{0.9} = 0.1027$...	$\hat{b} = 1.3435$ $\hat{m} = 0.9814$ $\hat{\eta} = 4.9511$ $t_{0.9} = 0.1057$

...
15000	$\hat{b}=14.926$ $\hat{m}=3.1104$ $\hat{\eta}=0.003$ $t_{0.9}=0.1514$	$\hat{b}=15.119$ $\hat{m}=0.0611$ $\hat{\eta}=0.983$ $t_{0.9}=1 \times 10^{-14}$	$\hat{b}=15.425$ $\hat{m}=0.0736$ $\hat{\eta}=1.877$ $t_{0.9}=1 \times 10^{-11}$	$\hat{b}=15.162$ $\hat{m}=0.0916$ $\hat{\eta}=2.909$ $t_{0.9}=6 \times 10^{-9}$...	$\hat{b}=15.165$ $\hat{m}=0.3209$ $\hat{\eta}=13.505$ $t_{0.9}=1.2612$

Build a three-dimensional graphic as shown in Figure 4.

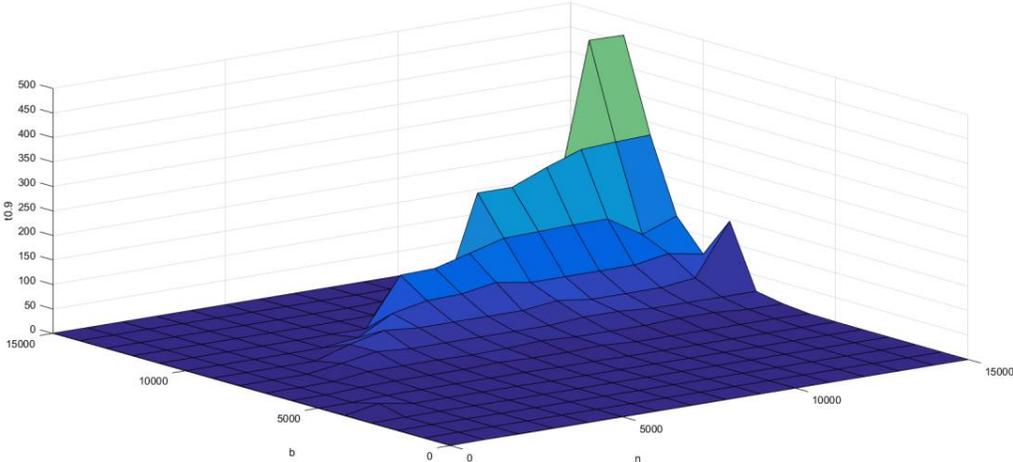


Fig 4: Construction of “Data Island”

At this point, the construction of the “data island” is completed. Each Z-axis value determined by the X-axis and Y-axis values is like a small island, so this method is named “Data Island”. As mentioned above, the optocoupler has $t_{0.9} > 10$, so $t_{0.9} = 10$ can be compared to “sea level”, while the “island” $t_{0.9}$ can be regarded as its “altitude”. When taking $t_{0.9} = 10$, because the “altitude” of some “islands” is too low, it will be submerged by “sea level”, as shown in Figure 5.

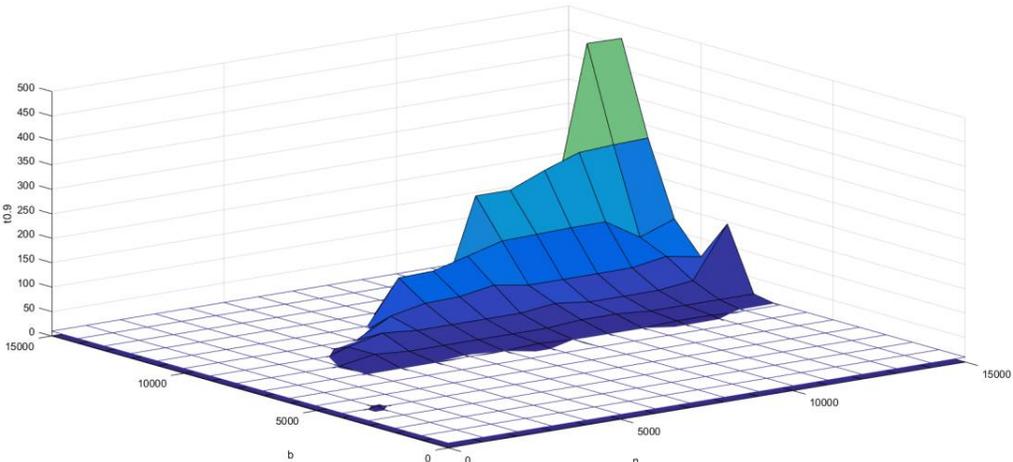


Fig 5: “Data Island” with “Elevation” at 10

As shown in the above figure, when the “sea level” is 10, many “islands” are inundated, and these “submerged” “islands” are the points that need to be eliminated. In the “islands” that are not

submerged, the parameter \hat{m} is discarded if it is estimated greater than 10. Since the reliability estimates are conservative, we look for the “islands” closest to “sea level” in islands that are not “submerged”. This allows us to determine a relatively accurate estimate of $t_{0,9}$ and determine the approximate range of the initial value.

4). “Island” search and elimination

The appropriate “island” is generated in data points where the estimated $t_{0,9}$ is greater than or equal to 10. These “islands” are examined in two steps.

First, approximation of m value. Observing these data points, we can find that most of the m values are less than 5, so let $m=5$, and then solve these data points. Due to space limitations, the parameter estimates for the “islands” that are not submerged when $m=5$ are not given here. When $m=5$, the “altitude” of “island”, that is, $t_{0,9}$, has changed, and some have “raised” and some “decreased”. Three “islands” were submerged by “sea level” due to the height drop, so these three points were eliminated. In addition, there is also an “island” that has been eliminated because it has a m estimated value of more than 10. Continue to observe the remaining estimates, it can be seen that the m value of most “islands” floats around 2, so take $m=2$, and then solve the relevant parameter estimates. Due to space limitations, the estimated values obtained are not given. By observing these values, it can be seen that the “altitude” of some “islands” has changed, and there are two cases of ascending or descending. But no “islands” have fallen below “sea level”. It can be known at present that the minimum value of $t_{0,9}$ is 10.121, and the question now concerned is whether it is the minimum point sought. Therefore, the next step is to verify it.

Second, the verification of the maximum iteration number solution and the convergence solution. As mentioned above, the mathematics software uses the Newton-Raphson method. It relies on the initial value. If the initial value is not set properly, the maximum iteration number solution is reached, that is, the convergence solution is not obtained. A theorem is given below:

Theorem: If $f(x)$ is continuous and the zero point to be solved is isolated, then there is a region around the zero point, and the Newton iteration must converge as long as the initial value lies within this neighborhood [28].

This theorem means that if the Newton iteration method does not converge, the initial value must not be in this region. Based on this, the parameter estimates \hat{b} , \hat{m} and $\hat{\eta}$ obtained when $m=2$ are returned to the initial value, and it is observed whether the equation can obtain a convergent solution. If the convergence solution is not obtained, the previously obtained parameter estimate is the maximum iteration number solution. It should be pointed out that Matlab can judge whether the solution converges by itself. If the word “fsolve stopped because it exceeded the function evaluation limit, options. MaxFunctionEvaluations = 300 (the default value).” appears, it is proved that the maximum number of iterations is reached, and the parameter estimation does not converge the equation. If the words “fsolve completed because the vector of function values is near zero as measured by the default value of the function tolerance, and the problem appears regular as measured by the gradient.”, it is proved that the iterative method converges, and the parameter estimation value can make the equation converged. On the other hand, as mentioned

above, the disadvantage of the Newton iteration method is that it is highly dependent on the initial value, and once the initial value deviates too far from the analytical solution, no convergence result is obtained. On the contrary, if the initial value is close to the analytic solution, the result of convergence must be obtained. This means that if the parameter estimates \hat{b} , \hat{m} and $\hat{\eta}$ are returned to the initial value, if the newly obtained solutions \hat{b}_1 , \hat{m}_1 and $\hat{\eta}_1$ are extremely small compared to before, then \hat{b} , \hat{m} and $\hat{\eta}$ necessarily converge the equation. In other words, \hat{b} , \hat{m} and $\hat{\eta}$ are in the same small convergence interval as \hat{b}_1 , \hat{m}_1 and $\hat{\eta}_1$, and both sets of solutions will converge the equation. If two sets of parameter values are used to find $t_{0.9}$, the change in $t_{0.9}$ will be small, that is, no drastic changes will occur. There are 7 groups of data in the software solution process, the first type of words pops up, thus the maximum number of iterations is reached, and other 43 groups of data all pop up the second type of words, that is, all convergence, so the seven groups of data are eliminated. In addition, it can be seen that the changes in the $t_{0.9}$ values of the 7 sets of data are very intense, while the changes in the $t_{0.9}$ values of the remaining data are extremely weak. The $t_{0.9}$ values (43 in total) generated by the remaining data are sorted from small to large. Since the maximum and minimum values are very different, the point of interest is the minimum point. Therefore, for the convenience of observation, take about the first 60% of the data points, and draw a graph as shown in Figure 6.

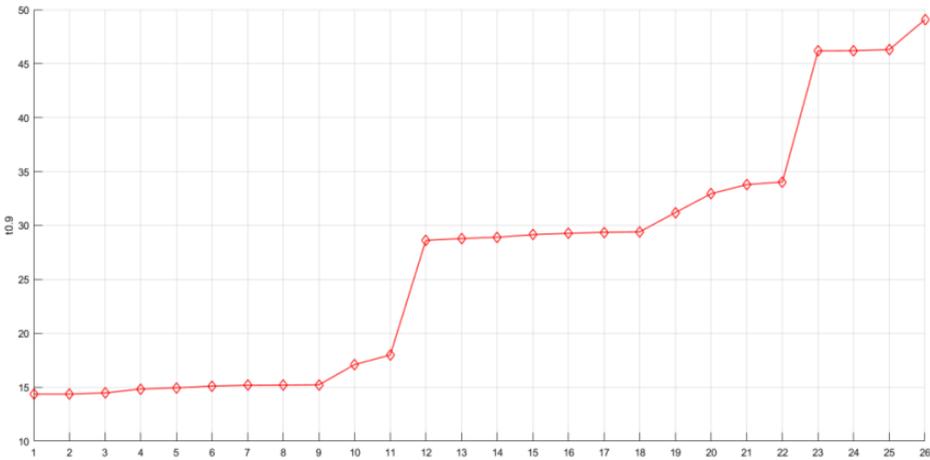


Fig 6: The trend of $t_{0.9}$ obtained by part convergent solutions

As shown above, the convergence of the $t_{0.9}$ value obtained by the convergence solution is about 14.3 to 15.1 years, which are 9 points. As the value of $t_{0.9}$ increases, the value of $t_{0.9}$ obtained by the convergence solution becomes more and more dispersed. If we make a conservative estimate, we can take the minimum value directly, or take the average of the parameter estimates of the 9 points, and then find $t_{0.9}$, about 14.6 years.

5) Determination of initial value and $t_{0.9}$. The Newton-Raphson method was used to eliminate and select the “islands”, but since the solution solutions are locally optimal and there is no global optimization, the Newton-Raphson method can only indicate the minimum value of $t_{0.9}$ and the

distribution trend of the convergence solution, which can be satisfied if a conservative assessment is made. But this may not be objective, so based on the Newton-Raphson method, return to starting point, the maximum likelihood estimation, using the convergence solution to solve $\ln L$, the maximum value of the log-likelihood function. Obviously, the convergence solution that maximizes $\ln L$ and the $t_{0.9}$ obtained by the convergence solution are the final desired results. As described above, since \hat{b} , \hat{m} and $\hat{\eta}$ and \hat{b}_1 , \hat{m}_1 and $\hat{\eta}_1$ are in extremely small difference, the obtained $\ln L$ values are also extremely small. Therefore, the $\ln L$ values obtained by substituting \hat{b} , \hat{m} and $\hat{\eta}$ and \hat{b}_1 , \hat{m}_1 and $\hat{\eta}_1$ are plotted in Figure 7.

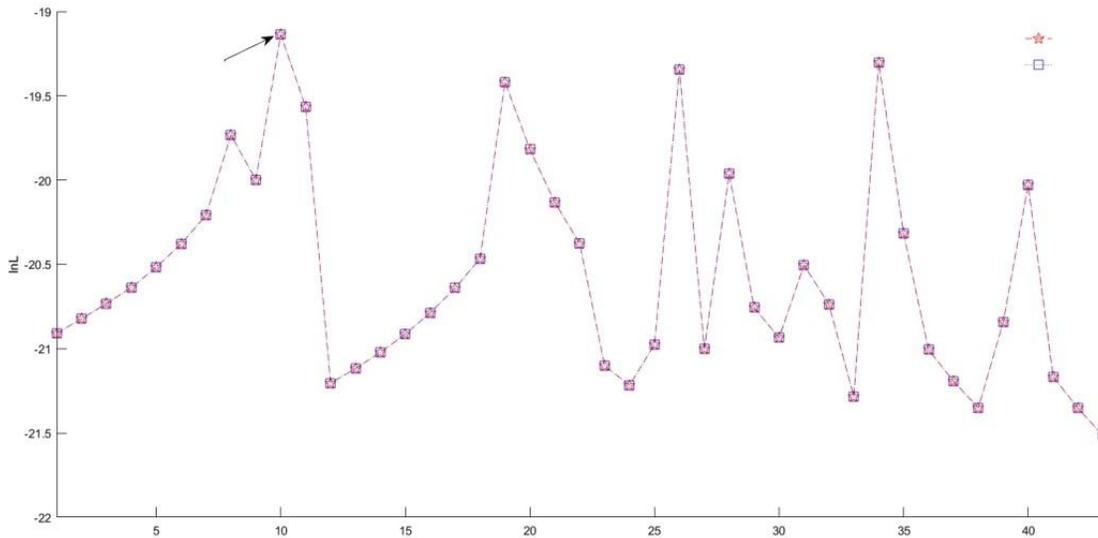


Fig 7: $\ln L$ values obtained from different parameter values

As shown in the figure above, the arrow points to the maximum point of $\ln L$. The orange five-pointed star represents the $\ln L$ value obtained using $(\hat{b}, \hat{m}, \hat{\eta})$, and the blue square represents the $\ln L$ value obtained using $(\hat{b}_1, \hat{m}_1, \hat{\eta}_1)$, and it can be seen that the $\ln L$ values obtained by the two are almost identical. So far, a reliable estimate of the relevant parameters and $t_{0.9}$ has been obtained using the “data island” method, ie $(\hat{b}_1, \hat{m}_1, \hat{\eta}_1)$ is (8661.2, 3.73, 5560.7), $t_{0.9} \approx 17a$. Of course, the initial value can also be considered as $(\hat{b}, \hat{m}, \hat{\eta})$, that is, (8660.8, 3.7303, 5559.8), and $t_{0.9}$ is still about 17 years.

6.2 Storage Reliability Under Sudden Failure Condition

According to the calculation results above, taking $b=8661.2$, $m=3.73$, $\hat{\eta}_h=5560.7$ hours, the following acceleration equation can be obtained:

$$\ln \eta_i = -16.5175 + \frac{8661.2}{T_i}. \quad (34)$$

By the formula (32), the characteristic lifetime of the photocoupler under normal temperature stress can be found to be 267866.9 hours (about 31.003 years). Therefore, the reliability of the

optocoupler under normal temperature stress (25 °C) is:

$$R_2(t) = 1 - F(t) = \exp\left[-\left(\frac{t}{31.0031103}\right)^{3.73}\right]. \tag{35}$$

Through the above formula, the sudden failure reliability curve of the optocoupler can be drawn.

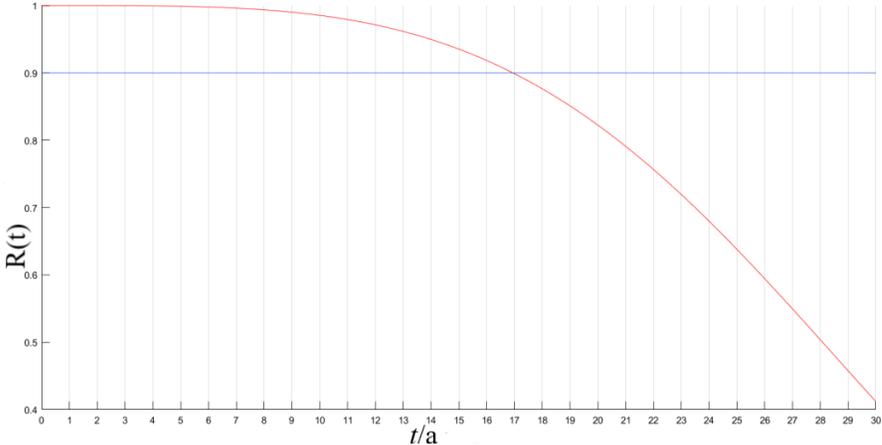


Fig.8 Storage reliability curve of sudden failure at 25 °C

It can be seen from the above figure that under the condition of sudden failure, the storage reliability of the optocoupler is greater than 0.9 before 17 years, and then the reliability shows a rapid decline.

6.3 Optocoupler Life Assessment in Storage Environment

Through the previous modeling and derivation, the reliability function and reliability curve of the optocoupler under degraded failure and sudden failure have been respectively obtained. As mentioned above, for independent competition failures, the reliability function is multiplied by the degradation and sudden failure reliability functions. By multiplying the equations (35) and (16), the reliability function of the optocoupler under normal temperature stress can be obtained. Since the reliability function of degradation has multiple curves with the threshold movement, the reliability functions of different thresholds are multiplied by the reliability function of sudden failure. The reliability curves are shown in Figure 9. For ease of comparison, the sudden failure reliability curve and the degradation failure reliability curves are also plotted in Figure 9.

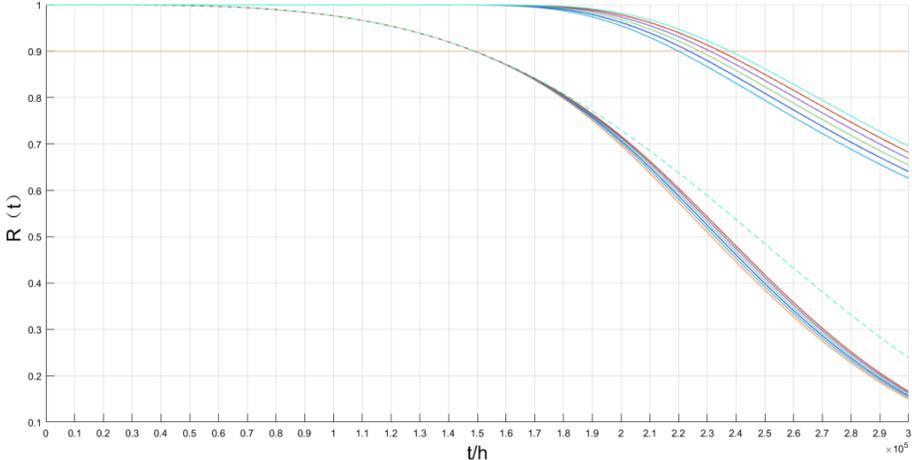


Fig 9: Total reliability curves of optocoupler storage

As shown in the above figure, the lowest curve group is the total reliability curve, the middle dotted line is the reliability curve of the sudden failure, and the right upper curve group is the degenerate failure curve. As can be seen from the above figure, whether it is degradation failure or sudden failure, the total reliability curves are obviously closer to the lower left than the reliability curves of the two, that is, the total reliability is less than the reliability of any failure mode. However, when the reliability is 0.9, and the reliability is greater than 0.9, the curve group of the total reliability and the sudden failure reliability curve are almost completely coincident. When the reliability is lower than 0.9, starting from about 0.8, as the reliability decreases, the total reliability curve group begins to gradually deviate from the reliability curve of the sudden failure, and slowly approaches to the lower left. Since $t_{0.9}$ is generally used as the reliability index, the storage life of the optocoupler is $147649 \text{ hours} = 17.089 \text{ a} \approx 17 \text{ a}$, that is, given a reliability of 0.9, the storage life of the optocoupler is about 17 years.

VII. SUMMARY

For evaluating the reliability of long-term storage of optocouplers, the reliability of two-factor independent competition failure based on Wiener process-based degradation failure and Weibull-based sudden failure is carried out. The Wiener process reliability model constructed in this paper takes into account the individualized differences, randomizes the drift coefficients, and establishes the random variable Arrhenius model. This method is more consistent with the actual situation than the traditional method which does not consider the individual difference. Using the two-step MLE method, estimated values of unknown parameters can be solved to overcome the limitations of traditional estimation methods. The estimated value of c is 1.4951, obviously it is greater than 1, and it is concave degradation according to classification. If the degradation data of leakage current is directly regarded as linear data, the accuracy of the evaluation will inevitably be greatly reduced, therefore, the data is linearized to make it more consistent with the characteristics of the Wiener process. Secondly, the solution to the reliability of Weibull distribution is studied.

Due to the lack of parameter estimates, on the basis of reasonable assumptions, the “data island” method is offered. The essence of "Data Island" is to use reverse thinking and the mathematical thought of partial exhaustive + gradual approximation to deduce the estimated value of each parameter step by step under the premise of setting the priori conditions. For the case of less data failure, it is necessary to set a priori condition. In the case of three or more stresses with failure data, the parameter value can be estimated with the estimated parameter value as the center, and the estimation range of the parameter value can be appropriately widened, and the required estimation value can be quickly found by using this method. According to the calculation in this paper, in fact η (that is η_h) the value range is too wide, which increases the calculation and workload. Through engineering experience, the upper limit of η (that is η_h) is often not greater than 10000. Therefore, it is also important to choose the appropriate scope. There are some extended assumptions about the "data island" approach:

(1) With the expansion of the value range of b and η_h , the convergent solution (that is, the convergent interval) becomes more and more sparse on the whole number line. However, the convergent solution always exists, and there is no case where the convergent solution (that is, the convergent interval) completely disappears after the value of b and η_h exceed certain points.

(2) From the properties of the maximum likelihood function and the practical meaning of the solution, the maximum value of $\ln L$ will always be at front of the $t_{0.9}$ ranking from small to large. No matter how the value ranges of b and η_h are extended, the position of the maximum value of $\ln L$ will not change, and the value of $\ln L$ will only get smaller and smaller (or the overall trend is smaller and smaller).

The above two conjectures are related to the further study of Weibull distribution-based maximum likelihood function, which is far from engineering practice and belongs to the research category of pure mathematical theory.

The shortcoming of this paper is that there are too few test samples, only 10. Although it meets the minimum number of test samples (≥ 7), it brings great trouble to solve the parameters of sudden failure. Of course, test samples are scarce because of their specific uses. However, in the future work, we still need to obtain more samples, which is the cornerstone of evaluating the accuracy of the conclusion. Otherwise life expectancy is a fancy but unrealistic number game.

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